A Position-Modulated Alignment Test Technique in the Presence of Ship's Flexure

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This article proposes a test procedure that is expected to reduce the effect of ship's flexure on test accuracy. The recommended procedure takes repeated measurements in alternating positions and averages the results. A test analysis is presented which derives an expression for test accuracy in terms of appropriate variables and the statistical characteristics of ship's flexure, assuming that the ship's flexure is a random walk. The expectation is that this analysis will enable the test designer to bound the time required to perform the test. Reasonable test parameters are assumed, and it is shown that they result in a practicable test time.

Introduction

THIS paper presents an alignment test that will improve the accuracy of alignment measurement in the presence of ship's flexure. Although the test method proposed has general applicability, it is applied specifically to the measurement of the misalignment between the outer axis and the accelerometers mounted on a stable platform using the system resolvers.

It is desirable to devise a test that can be performed in situ after installation of the guidance-system Inertial Measurement Unit (IMU). Assuming that there are at least two IMUs at the same installation, a simple test involves using one system as a reference to monitor the ship's motion during calibration of an adjacent system. The assumption that there is no relative motion between the two systems due to ship's flexure or twist is implicit in this technique. If the ship's flexure is shown to be small, then testing can be simplified greatly.

However, it appears that no very useful body of experimental information exists on this subject. This paper describes a test technique that enables the desired misalignment measurement to be made in the presence of a credible amount of ship's flexure. It appears possible that, assuming the twist is a random walk, even if the ship twists as much as 30 arcsec during the test, the measurement can still be made in a little over half an hour.

A more detailed investigation of the proposed test technique is necessary to establish its applicability for measuring misalignment in the presence of ship's flexure. However, the results for the simplified case considered herein show definite promise and justify further detailed investigation.

Description of Test

Using the resolver and the accelerometers, the stable member may be positioned in various ways to obtain a measurement of the outer-axis-to-accelerometer misalignment. At the same time, an adjacent IMU can be oriented to obtain compensating information about the ship's motion (both rotation and translation). By combining data from both IMUs, the effect of ship's motion can be eliminated.

Problems arise due to the effect of ship's flexure during the test, which corrupts the measurements. The object is to devise a test that can measure the misalignments adequately, using the measurements made in these various positions, in the presence of some amount of ship's flexure.

The desirable simple resolver tests may use two or more positions to make enough measurements to compute the misalignment. However, they all may be shown equivalent mathematically to the form shown in Eqs. (1) and (2). For convenience, these two equations will be called the "up" measurement and the "down" measurement. This does not imply that there need be only two positions or that they are actually up and down. It is merely a convenient manner of speaking:

$$M_U = B + x + \eta_{BU} + N_U \tag{1}$$

$$M_D = B - x + \eta_{BD} + N_D \tag{2}$$

where B is a constant term, or the mean of a randomly varying term; x is the desired term (misalignment); M_U is the up measurement; M_D is the down measurement; η_B is the unknown change in the constant, presumed to be a random walk; and N is the noise in the measurement introduced by the measurement instrument, presumed to be the accelerometer. Now if $n = N = 0$, then it is obvious that x may be found by combining Eqs. (1) and (2) to give

$$x = (M_U - M_D)/2 \tag{3}$$

Thus, the misalignment can be measured by making one up measurement and one down measurement. However, this may not be possible to the accuracy required in the presence of noise. One solution is to borrow a page from communication theory and use the idea of modulation. If the noise is presumed to have the typical power spectral density (PSD) curve (shown in Fig. 1), where the low-frequency noise is proportional to inverse frequency and the high-frequency noise is proportional to frequency, then, by modulating at the frequency (f_0) corresponding to the minimum noise, the noise in the signal is minimized.

Applying this idea to the problem of measuring misalignment accurately in the presence of ship's twist, the method that suggests itself is to cycle the up and down positions many times and average the measurements. Thus, the effect of a simple step change in the constant during the course of the test on the misalignment measurement will be reduced by the number of cycles used for the test. That is, if the ship twists 10 arcsec in one step, and 10 cycles of up-down measurements are made, then the resulting estimate error for x will be 10/10, or 1 arcsec.

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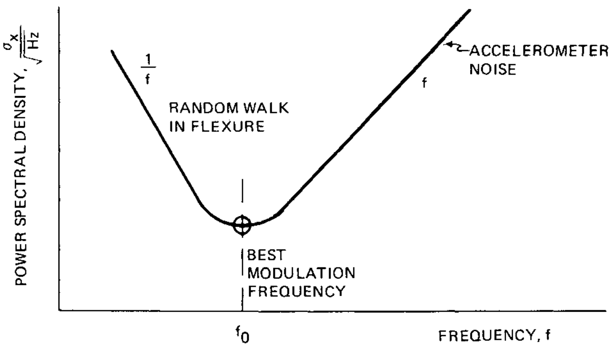

Fig. 1 Postulated power spectral density of test error.

With this in view, the following estimator was defined:

$$X = \sum_i^p (M_U - M_D)_i \Big/ 2p \tag{4}$$

where X is the random variable, which estimates the desired misalignment, and $i = 1,2,3,..., p$ which represents the number of up-down cycles.

By combining Eqs. (1) and (2) with Eq. (4), using the property of a random walk and equations from Ref. 1 for the statistics of the curve-fitting process used when collecting the accelerometer data, the following equations for the test error statistics can be developed (see Appendix):

$$\sigma_x^2 = \frac{48}{g^2}\left(\frac{\sigma_v^2 \Delta T}{T_{\text{tot}}^3}\right)p^2 + \frac{QT_{\text{tot}}}{8p^2} \tag{5}$$

where σ_v^2 is the variance of the accelerometer noise assumed Gaussian (ft/s)^2; ΔT is the collection time per point (from the accelerometer), in seconds; T_{tot} is the total test time, in seconds; Q is the random walk variance coefficient (arcsec^2/s); p is the number of up-down cycles; and σ_x^2 is the variance of the estimate of the misalignment (arcsec^2), which represents the accuracy of the test. The details of the derivation are presented in the Appendix.

Note the form of Eq. (5), which is a declining and then increasing function of p. The analogy to the PSD becomes clearer if Eq. (5) is written in the form of frequency by using the definition

$$f_m = p/T_{\text{tot}} \tag{6}$$

Then Eq. (5) becomes

$$\sigma_x^2 = \frac{48}{g^2}\frac{\sigma_v^2 \Delta T f_m^2}{T_{\text{tot}}} + \frac{Q}{8T_{\text{tot}}f_m^2} \tag{7}$$

By dividing both sides of Eq. (7) by a unit bandwidth β, it can be converted into a PSD form, and it is obvious that it has the shape of Fig. 1, as expected.

Equations (5) and (7) have a minimum that can be obtained by differentiating with respect to p (or f_m). The result is

$$\left.\begin{aligned} &\sigma_x^4 = (24/g^2)(Q\Delta T\sigma_v^2/T_{\text{tot}}^2) \\ &p = \sqrt{QT_{\text{tot}}}/2\sigma_x \end{aligned}\right\} \text{ at minimum } \sigma_x^2. \tag{8}$$

Equation (8) can be used as a set of design equations to investigate the test parameters. The next section undertakes a numerical investigation of possible combinations of test parameters.

A Study of Some Test Designs

This section uses the results of the derivation given in the Appendix and repeated in Eqs. (5), (7), and (8). Based on engineering judgment, values are chosen for the various terms in the equations that are believed to bound the problem and the resulting test time is obtained. To this end, some charts have been prepared to permit rapid evaluation of changes in some of the terms.

Equation (8), has been rearranged as follows:

$$p = \sqrt{QT_{\text{tot}}}/2\sigma_x \qquad T_{\text{tot}} = (1/\sigma_x^2)\sqrt{K}Q \qquad K = (24/g^2)\Delta T\sigma_v^2 \tag{9}$$

Reasonably conservative values of ΔT and σ_v are

$$\Delta T = 0.1 \text{ s} \qquad \sigma_v = 0.05 \text{ ft/s}$$

Then, using Eq. (9),

$$K = 5.79 \times 10^{-6} \text{ s}^3$$

With these values, one can draw Fig. 2. This plots total test time (T_{tot}) vs number of cycles (p). On it lie lines of constant Q and σ_x. Note that f_m depends only on T_{tot}, p, and Q, so that a line of constant Q is also a line of constant f_m, and is so marked in Fig. 2.

From this chart, one can seek a particular set of solutions. Choose a reasonable desired value of σ_x. The choice made here is $\sigma_x = 0.4$ arcsec. Then, along a line of constant σ_x, a family of solutions for T_{tot}, p, and Q are defined fully. In Fig. 3 just the line of constant $\sigma_x = 0.4$ is shown, and the rest of the chart is suppressed. Now the problem is to determine a reasonable value of Q to find the corresponding number of cycles and the total test time.

Up to now, the values assumed (ΔT and σ_v) have a sound basis of engineering experience. What value of Q should be chosen? This term is the random-walk coefficient defined by

$$\sigma_B^2 = Qt \tag{10}$$

where t denotes the time, Q the constant, and σ_B^2 the variance of the random-walk motion of the ship's flexure or twist.

Recall that the ship's flexure is modeled as a random walk rather than a Gaussian random process (white noise) because physically one would expect a gradual creep rather than a rapid jumping around. Then, define the following thought experiment: If one presumes an experiment that measures the ship's flexure between two adjacent IMUs, then after 2000 s (33 min), how much can one expect the ship twist to have "walked" from its initial value? A number like 10 arcsec seems credible. Then

$$Q = (10)^2/2000 = 0.05 \text{ arcsec}^2/\text{s}$$

Similarly, if the number chosen is 100 arcsec, then the value of Q is 5. A twist value may be assigned to each value of Q. This is marked on the chart in Fig. 3. Thus, a range may be chosen that seems to encompass all possible extremes. The line in Fig. 3 shows such a range, from $Q = 0.01$ to 5, which goes from an expected motion of 4.47 to 100 arcsec. Now, looking at these extremes, observe that the total test time ranges from 320 to 6800 s, with the required number of cycles from 2.2 to 240.

The point of the preceding discussion is that, for a desired value of σ_x equal to 0.4 arcsec, the required total test time is practicable, even at the extremes. For a "reasonable" value, $Q = 0.05$, then the value of T_{tot} is 700 and $p = 7.5$, a very reasonable combination. Pursuing this design, Fig. 4 shows a plot of σ_x vs p from Eq. (5), using the previously mentioned design values. This shows the off-design performance and is important, since, obviously, p can take on only integer values in the real test. The graph shows that the trough of the curve is reasonably flat.

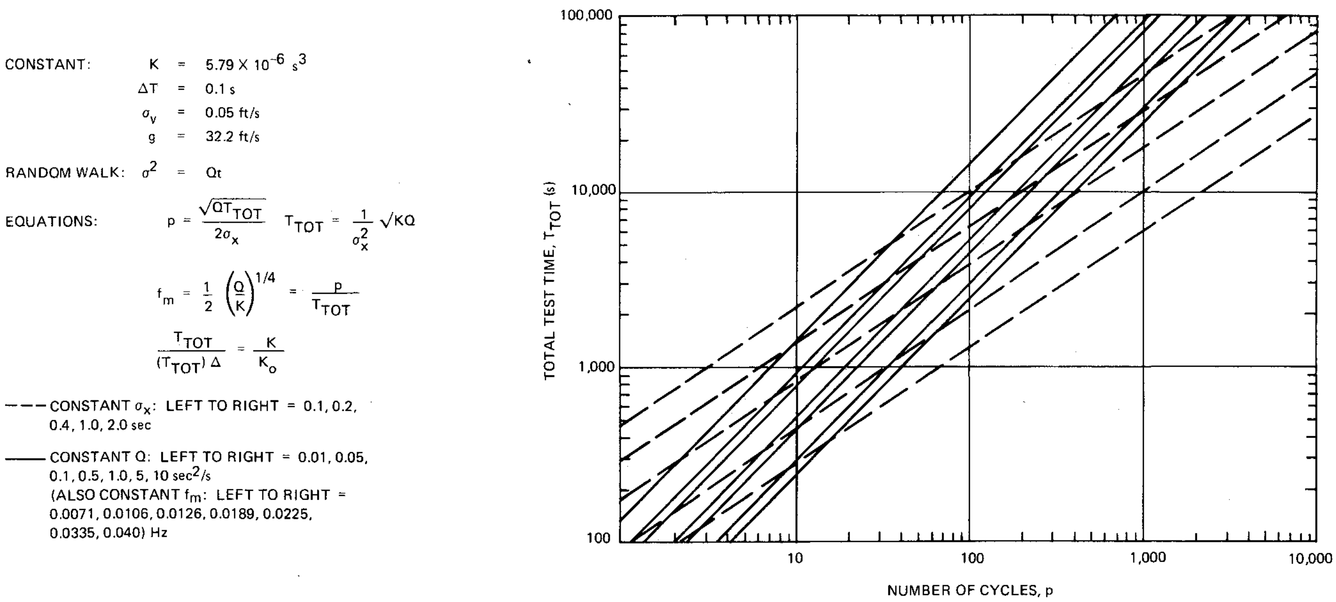

Fig. 2 Design chart for test.

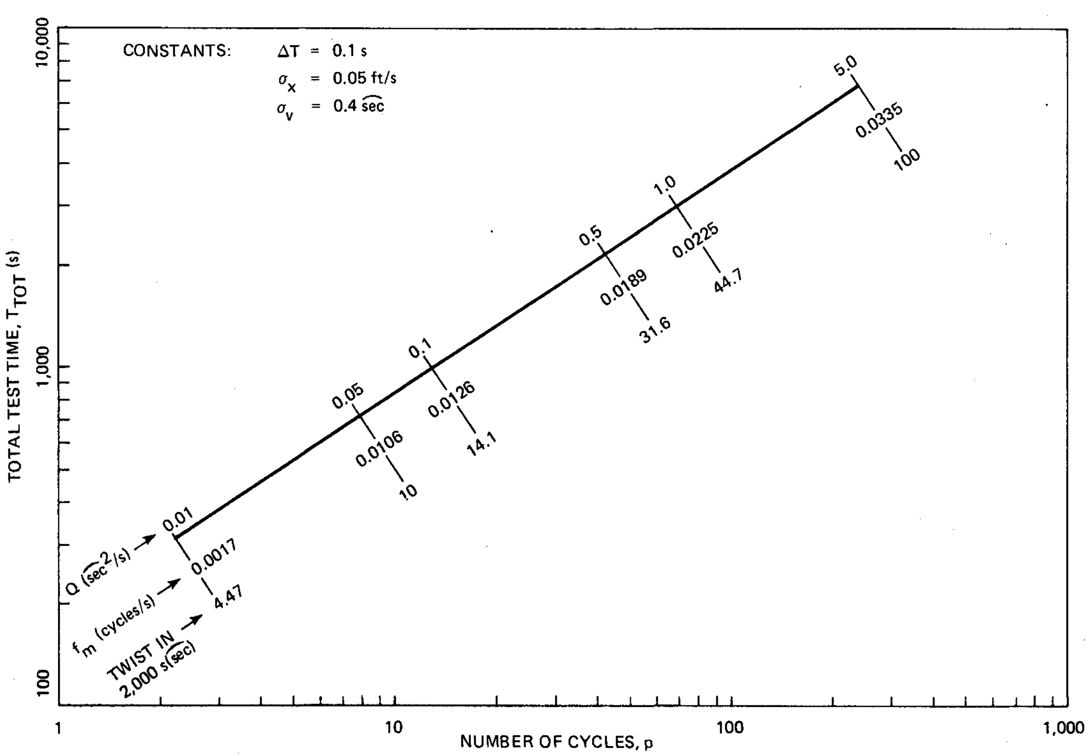

Fig. 3 Particular set of solutions for a given test error.

Table 1 shows some selected values from Fig. 3, and Table 2 shows some selected values from Fig. 2 with Q held constant.

Conclusions

The results presented show that a cyclic test technique is feasible and can be applied to misalignment measurement. The detailed numerical example for estimating the effect of ship's creep on the measurement shows the practicability of the proposed test technique and how the analysis can be used to bound the problem.

Appendix: Derivation of Test Error

This Appendix presents a derivation of the equations used to determine the test error. Recall that in the usual test of this kind the measurement is made via the integrating accelerometer using a least-squares fit to the output velocity data. The measurements thus made in two positions can be used to compute the desired parameters. This proceeds as follows:

$$M_1 = B + x \tag{A1}$$

$$M_2 = B - x \tag{A2}$$

$$x = (M_1 - M_2)/2 \tag{A3}$$

where M is the measurement, B the constant, and x the desired parameter. In the variation of the type of test proposed here,

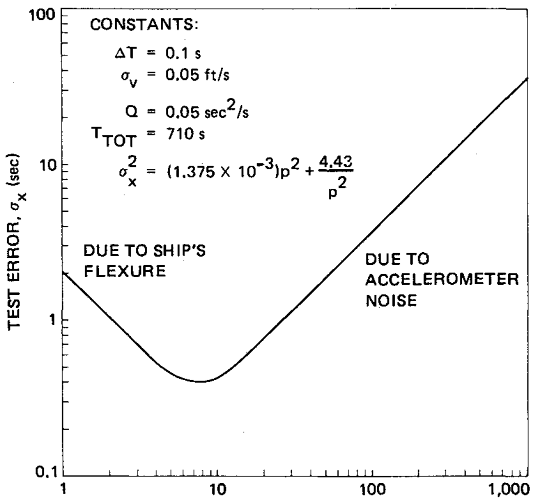

Fig. 4 Test error with fixed amount of flexure, Q, and varying p.

Table 1 Test parameters along $\sigma_x = 0.4$ arcsec, with $\Delta T = 0.1$ s and $\sigma_v = 0.05$ ft/s

Q, arcsec^2/s	0.01	0.05	0.5	5
Assumed twist in 2000 s, arcsec	4.47	10	31.6	100
p, cycles	2.2	7.9	42	240
T_{tot}, s	310 (5.16 min)	710 (11.8 min)	2150 (35.8 min)	6800 (113 min)

Table 2 Test time as a function of number of cycles for all other parameters fixed ($\Delta V = 0.1$, $\sigma_v = 0.05$, $Q = 0.05$, (from Fig. 2)

T_{tot}, s	100	700	3000	12,000
p, cycles	1	7.5	32	120
σ_x, arcsec	1	0.4	0.2	0.1

these measurements are made many times and then averaged to find x. More precisely, the equations can be written

$$V_U = B + x + \eta_U \tag{A4}$$

$$V_D = B - x + \eta_D \tag{A5}$$

where V_U, V_D are the up and down states, respectively, B is a constant, x the parameter of interest, and η the random-walk process. Note that although the terminology used here is "up" and "down," this merely is a convenient way of differentiating between two positions. In an actual test, there may really be an up and down position, or just two different positions.

The measurement M is

$$M_U = V_U + N_U \tag{A6}$$

$$M_D = V_D + N_D \tag{A7}$$

where N is the error in the measurement. This is assumed to be a random variable from a normal distribution. Then, the estimate for x is defined, analogous to Eq. (A3), as

$$X = \sum^{p} (M_U - M_D)_i \Big/ 2p \tag{A8}$$

for $i = 1,2,\ldots,p$. The number of pairs of measurements, or cycles, is p.

If a process is defined as the sum of independent random increments, then it is called a random walk. Let u be the process variable; then a recursive equation may be written for the state at any point

$$u_{n+1} = u_n + N_n \tag{A9}$$

where N is the random variable from a normal distribution. By starting at $n = 0$, and using the recursive relation [Eq. (A9)], it is easy to develop another form for u_n:

$$u_n = \sum^{n-1} N_i \tag{A10}$$

for $u_0 = 0$; $i = 1,2,\ldots,(n-1)$.

The basic equations have now been defined. By appropriate manipulation, the statistics of X will be obtained. Combining Eqs. (A4-A7) into Eq. (A8), for $i = 1,2,\ldots,p$,

$$X = \sum^{p} (x + N_U + \eta_U + x - \eta_D - N_D)_i \Big/ 2p \tag{A11}$$

$$X = x + \left\{ \sum^{p} [(\eta_U - \eta_D) + (N_U - N_D)]_i \Big/ 2p \right\} \tag{A12}$$

By using Eq. (A9), Eq. (A12) becomes

$$X = x + \left\{ \sum^{p} [(N_B + N_U - N_D)]_i \Big/ 2p \right\} \tag{A13}$$

where $i = 1,2,\ldots,p$, and N_B is the independent increment of the random walk.

Equation (A13) is now in the form where all random terms are independent random variables from a normal distribution with zero mean. Then, the expectation of x, $\bar{X}$, is

$$\bar{X} = x \tag{A14}$$

Thus, X is an unbiased estimate for the parameter x. Compute the expectation of X^2:

$$\overline{X^2} = \bar{X}^2 + (\sigma_a^2/2p) + (\sigma_B^2/4p) \tag{A15}$$

where σ_B^2 is the variance of the random variable N_B, and σ_a^2 the variance of the random variable N. Using the usual relationship $\sigma^2 = \overline{X^2} - \bar{X}^2$, the variance of X is

$$\sigma_x^2 = \overline{X^2} - \bar{X}^2 = (\sigma_a^2/2p) + (\sigma_B^2/4p) \tag{A16}$$

Then, Eq. (A16) is the desired expression for the test error (or variance) σ_x in terms of the test uncertainties. Note that σ_B^2 is the variance of a random walk, and σ_a^2 is the variance of a normal distribution. The variance of a random walk is

$$\sigma^2 = Qt \tag{A17}$$

The random-walk process has a variance which increases linearly with time. To apply Eq. (A17) to (A16), the time must be taken as equal to the time interval over which N_B is applied. This is the time at one position, denoted T. Then

$$\sigma_B^2 = QT \tag{A18}$$

and, substituting into Eq. (A16),

$$\sigma_x^2 = (QT/4p) + (\sigma_a^2/2p) \tag{A19}$$

The number of cycles p may be expressed in terms of the test time T_{tot} and the position time T as follows:

$$p = T_{tot}/2T \tag{A20}$$

Then, substituting into Eq. (A19),

$$\sigma_x^2 = (QT_{\text{tot}}/8p^2) + (\sigma_a^2/2p) \tag{A21}$$

Next, an expression for σ_a^2 must be obtained. From Ref. 1 or 2, the variance of the coefficient of the linear term of a first-order curve fit may be obtained in terms of the variance of the individual measurement. The appropriate equation is

$$\sigma_a^2 = \sigma_v^2 (12/g^2)(\Delta T/T^3) \tag{A22}$$

where σ_v^2 is the variance of one accelerometer measurement (ft/s); ΔT is the time between measurements; and T is the collection time. For this analysis, T is assumed to be the time in one position. Substituting Eq. (A22) into (A21) and using Eq. (A20) to eliminate T, the result is

$$\sigma_x^2 = \left(\frac{48}{g^2}\right)\left(\frac{\sigma_v^2 \Delta T}{T_{\text{tot}}^3}\right)p^2 + \frac{QT_{\text{tot}}}{8p^2} \tag{A23}$$

Equation (A23) is the desired expression for the test variance σ_x^2 in terms of the pertinent test parameters. Note that by defining a frequency

$$f_m = p/T_{\text{tot}} \tag{A24}$$

and substituting into Eq. (A23), the result is

$$\sigma_x^2 = \frac{48}{g^2}\sigma_v^2 \frac{\Delta T f_m^2}{T_{\text{tot}}} + \frac{Q}{8T_{\text{tot}} f_m^2} \tag{A25}$$

By defining a unit bandwidth β and dividing both sides of Eq. (A25) by β, the equation would then be exactly in the form of a power spectral density, with a $1/f_m$ dependence at low frequency, and an f_m dependence at high frequency. The low-frequency noise is due to the random walk, and the high-frequency noise is due to the Gaussian noise from the accelerometer measurement.

Turning back to Eq. (A23), it is obvious that there exists a minimum of σ_x with respect to p. This is found easily by differentiation, and is

$$(\sigma_x)_{\min} = \left[\left(\frac{24}{g^2}\right)Q\left(\frac{\Delta T \sigma_v^2}{T_{\text{tot}}^2}\right)\right]^{1/4}$$

$$(p)_{\min} = \mathrm{T}_{\text{tot}}\left[\left(\frac{g^2}{8\times 48}\,\frac{Q}{\sigma_v^2 \Delta T}\right)\right]^{1/4} \tag{A26}$$

Equation (A26) can be manipulated into many forms. Some of them are

$$p = \sqrt{QT_{\text{tot}}/2\sigma_x} \qquad T = \sigma_x\sqrt{T_{\text{tot}}/Q} \tag{A27}$$

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